

THE TRIGONOMETRIC MEASURE OF ANGLES

- *blending Pythagorus (circa 540 B.C.)
and Descartes (circa 1637)*

--> *taking a look at different quadrants
of the Cartesian Plane*

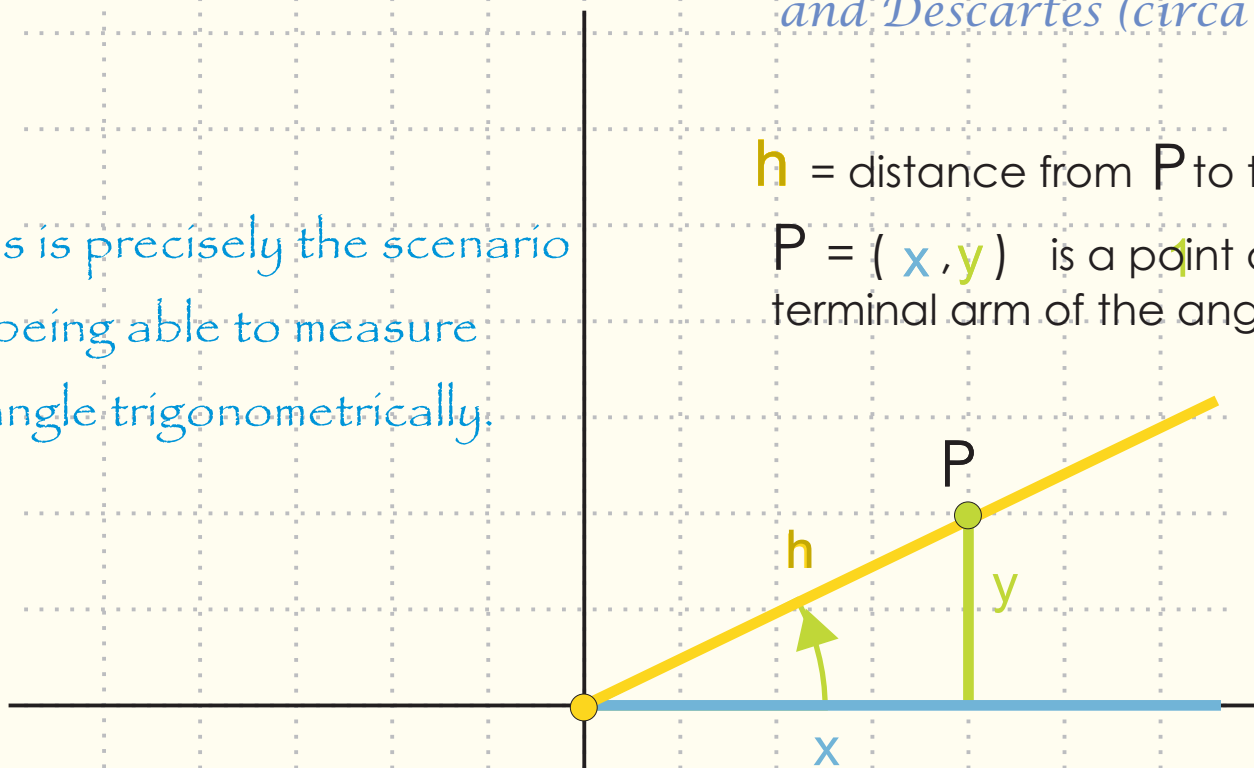
THE TRIGONOMETRY OF ANGLES

- *blending Pythagorus (circa 540 B.C.)
and Descartes (circa 1637)*

This is precisely the scenario
for being able to measure
an angle trigonometrically.

h = distance from P to the origin

$P = (x, y)$ is a point on the
terminal arm of the angle



Points on the terminal arm can
be given Cartesian coordinates.

The horizontal and vertical
coordinate values provide
a way of constructing a
right triangle which contains
the angle at one of its vertices.

STANDARD POSITION
OF AN ANGLE

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm

↻ Positive angle

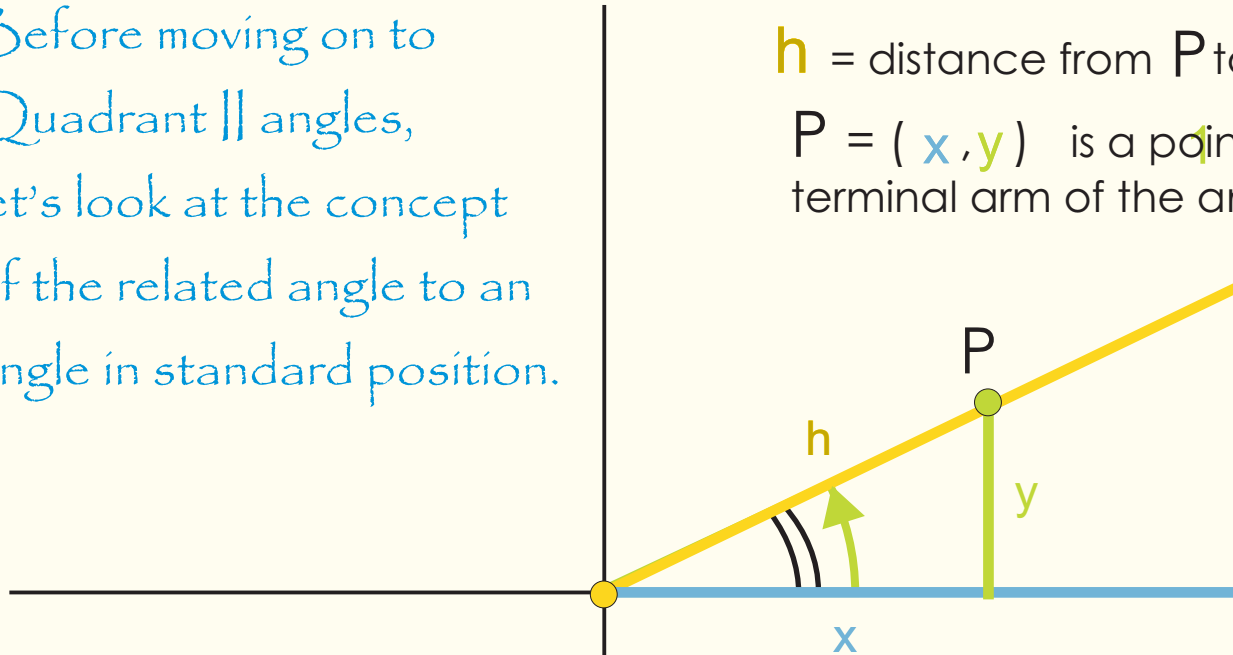
THE TRIGONOMETRY OF ANGLES IN QUADRANT I

- *blending Pythagorus (circa 540 B.C.)
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Before moving on to
Quadrant II angles,
let's look at the concept
of the related angle to an
angle in standard position.

h = distance from P to the origin

$P = (x, y)$ is a point on the
terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

STANDARD POSITION OF
A QUADRANT I ANGLE

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm
lies in quadrant I

// Related angle
to

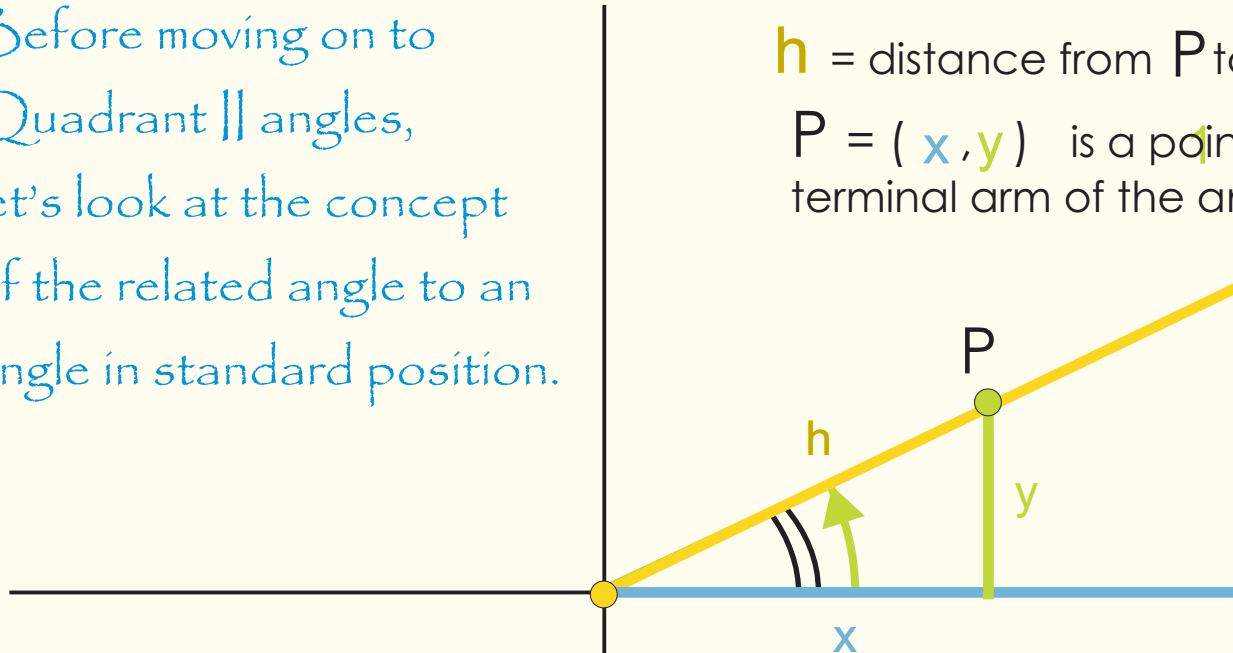
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Definition:

RELATED ANGLE TO AN ANGLE

The related angle to
is the acute angle
between its terminal arm
and the nearest extension
of the x-axis.

STANDARD POSITION OF A QUADRANT I ANGLE

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm
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// Related angle
to

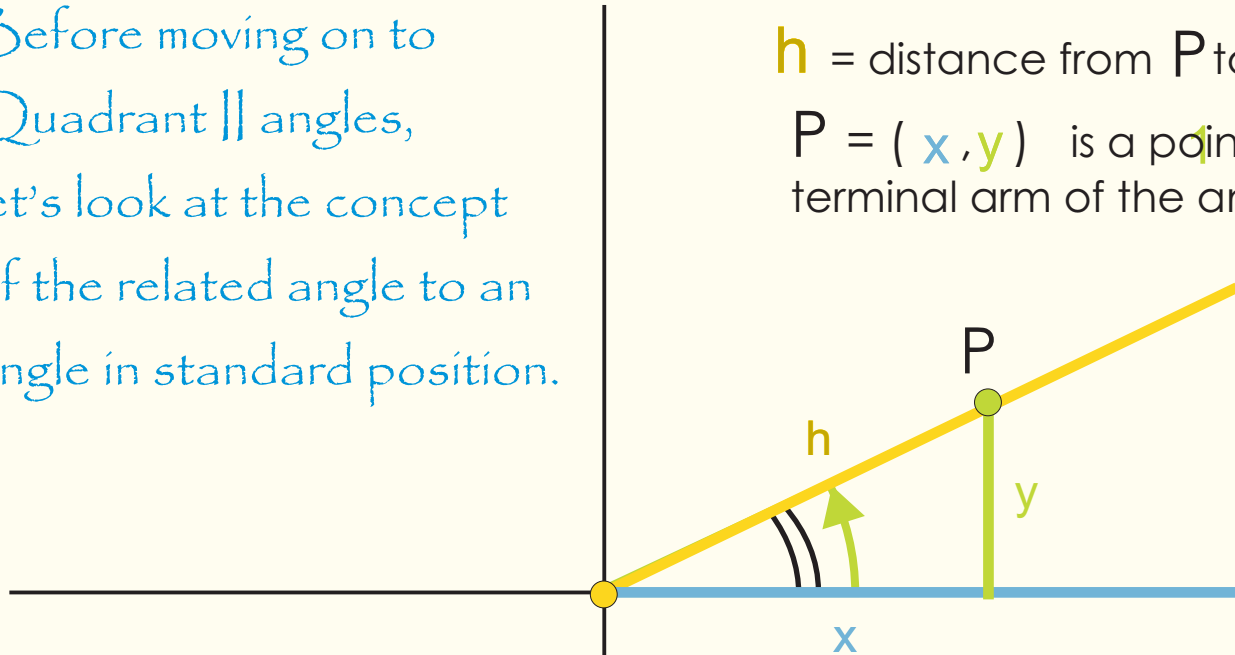
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Definition:

RELATED ANGLE TO AN ANGLE

The related angle to
is the acute angle
between its terminal arm
and the nearest extension
of the x-axis.

For first quadrant angles:

=

STANDARD POSITION OF A QUADRANT I ANGLE

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm
lies in quadrant I

// Related angle
to

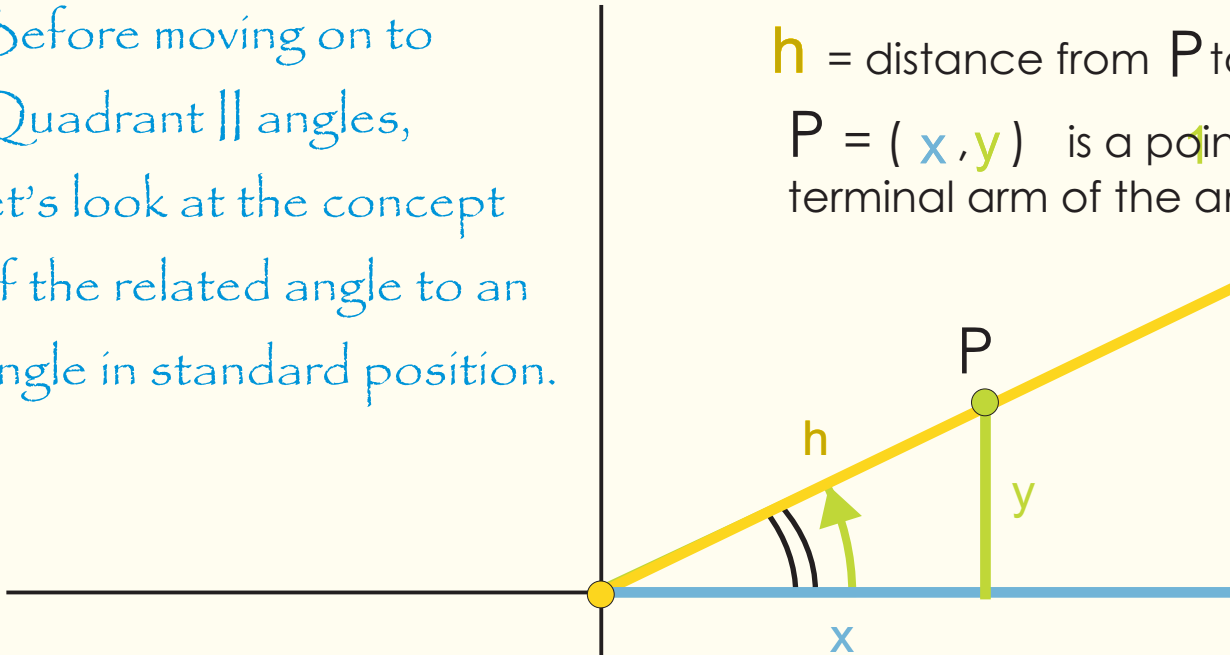
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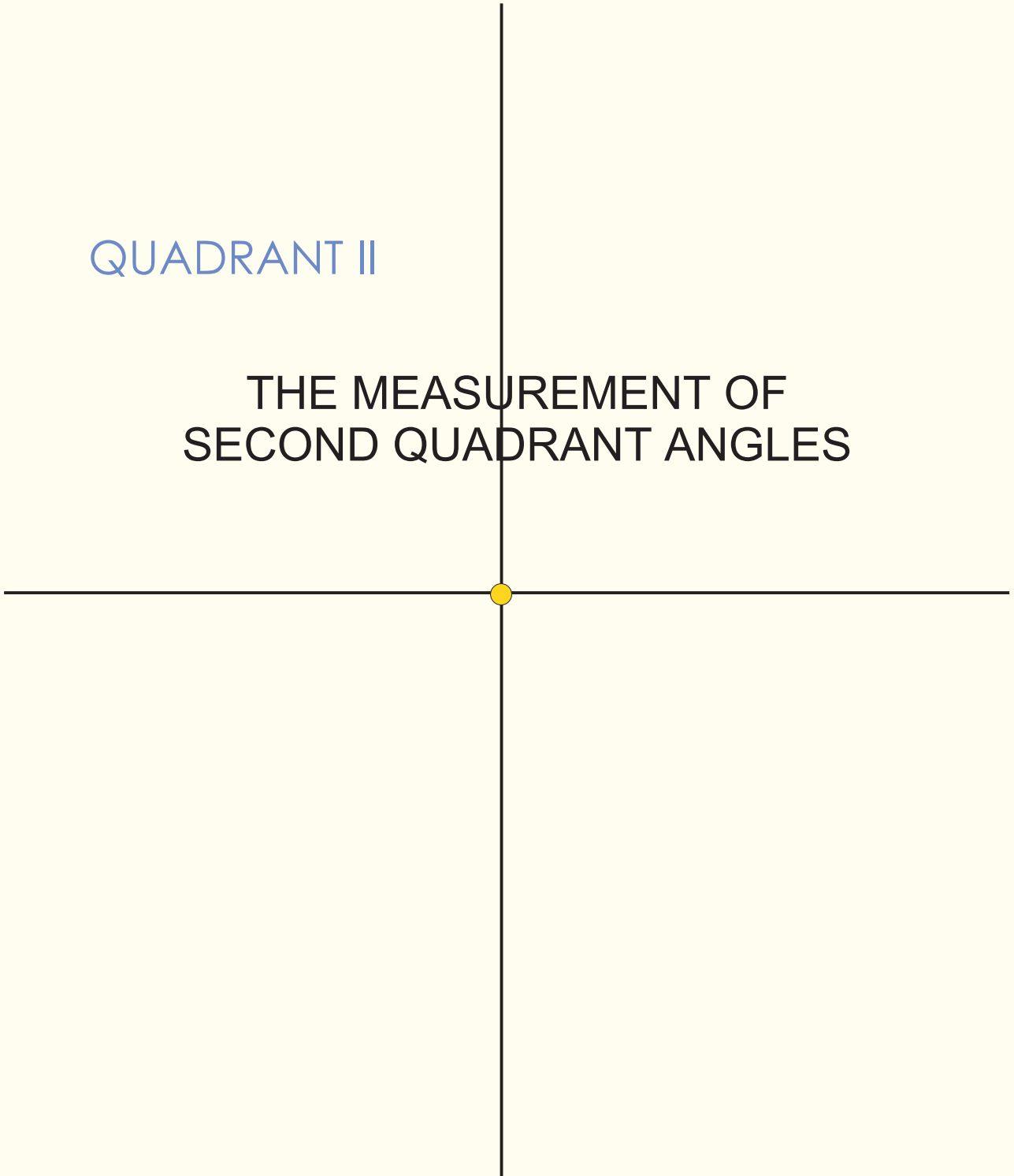


The trigonometric measures
of an angle in the FIRST
quadrant are measured using
the sides of the triangle in
the FIRST quadrant containing
the angle itself.

However, note that, in this case,
= its related angle .

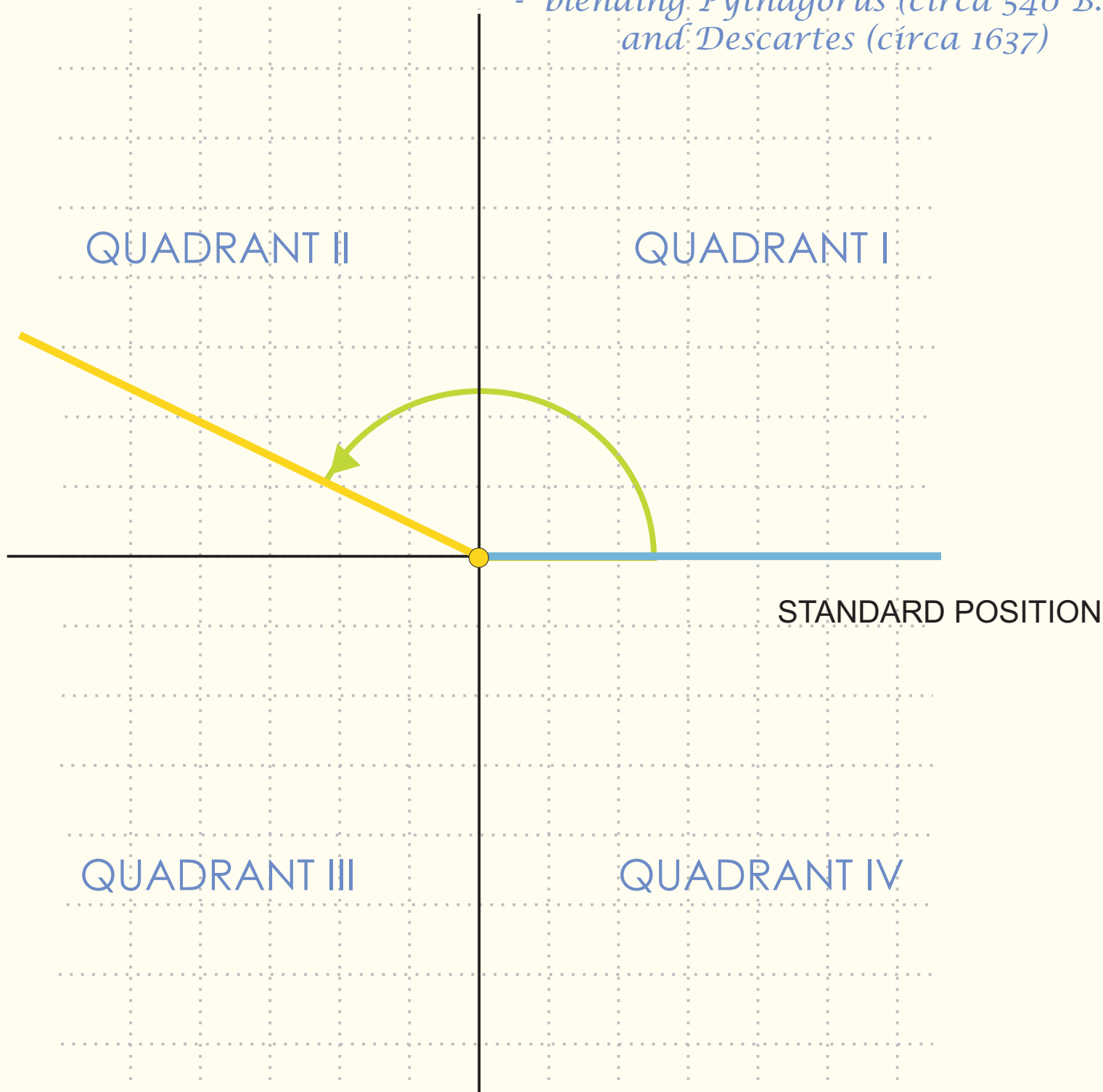
QUADRANT II

THE MEASUREMENT OF
SECOND QUADRANT ANGLES



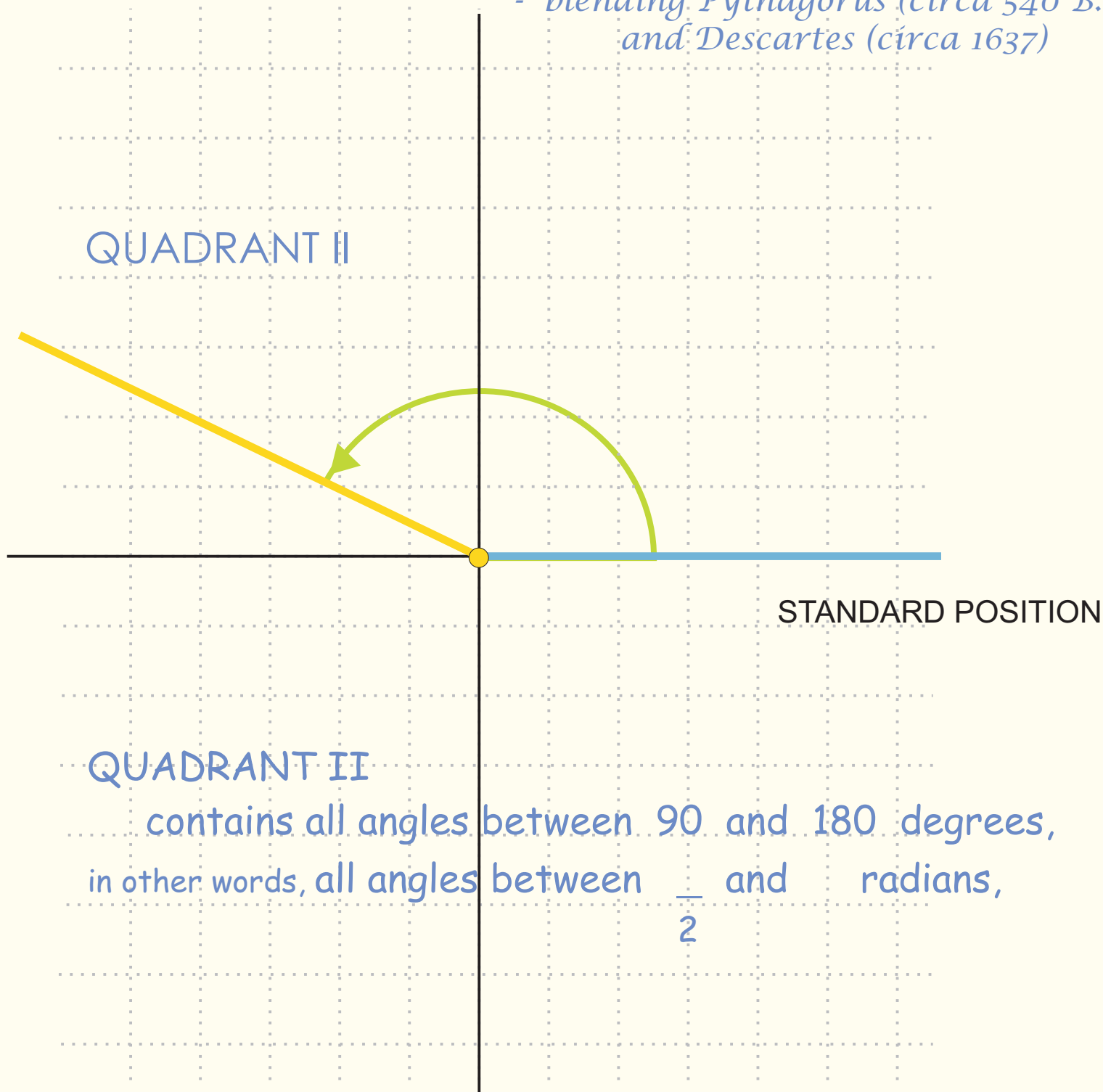
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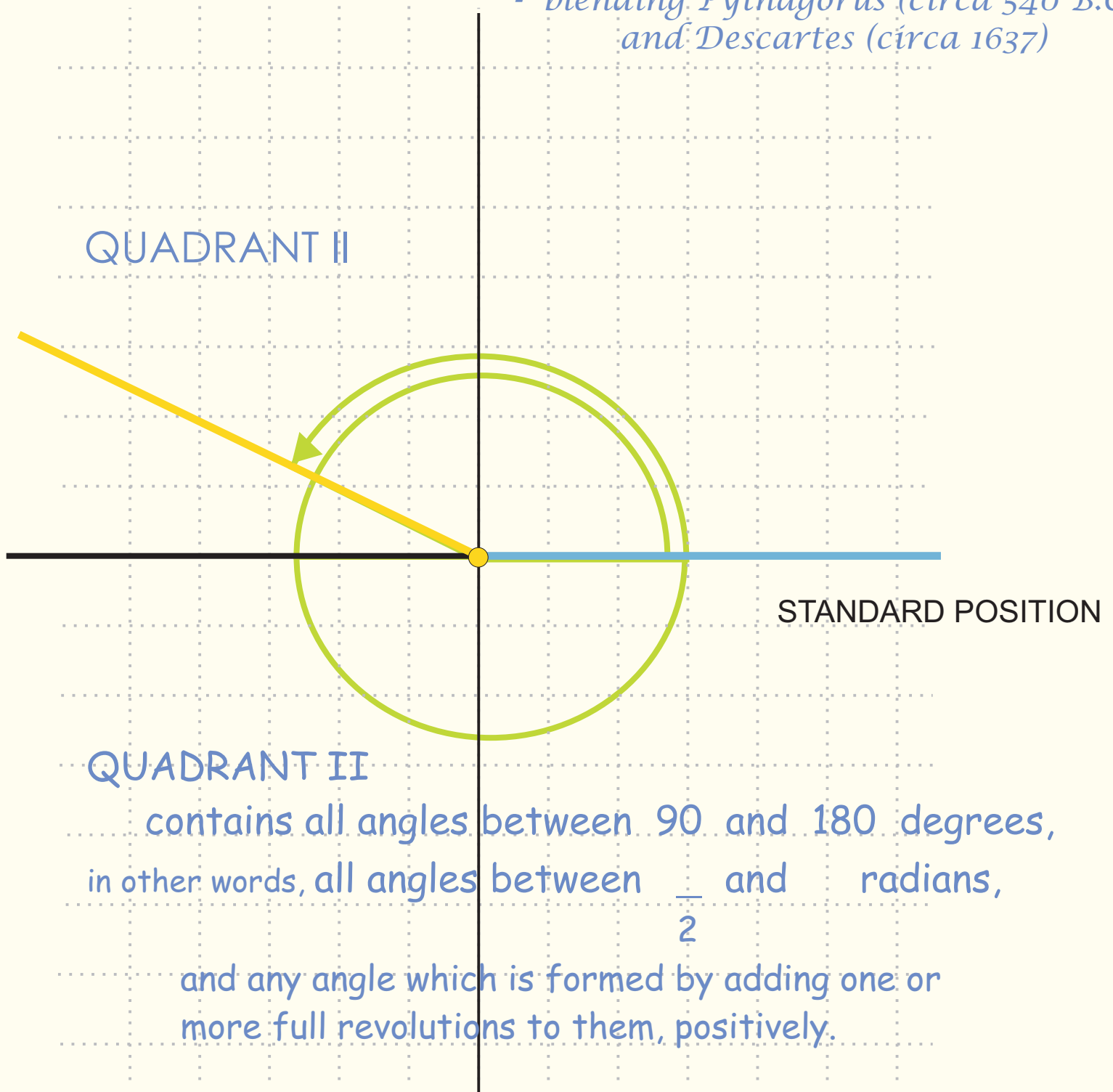
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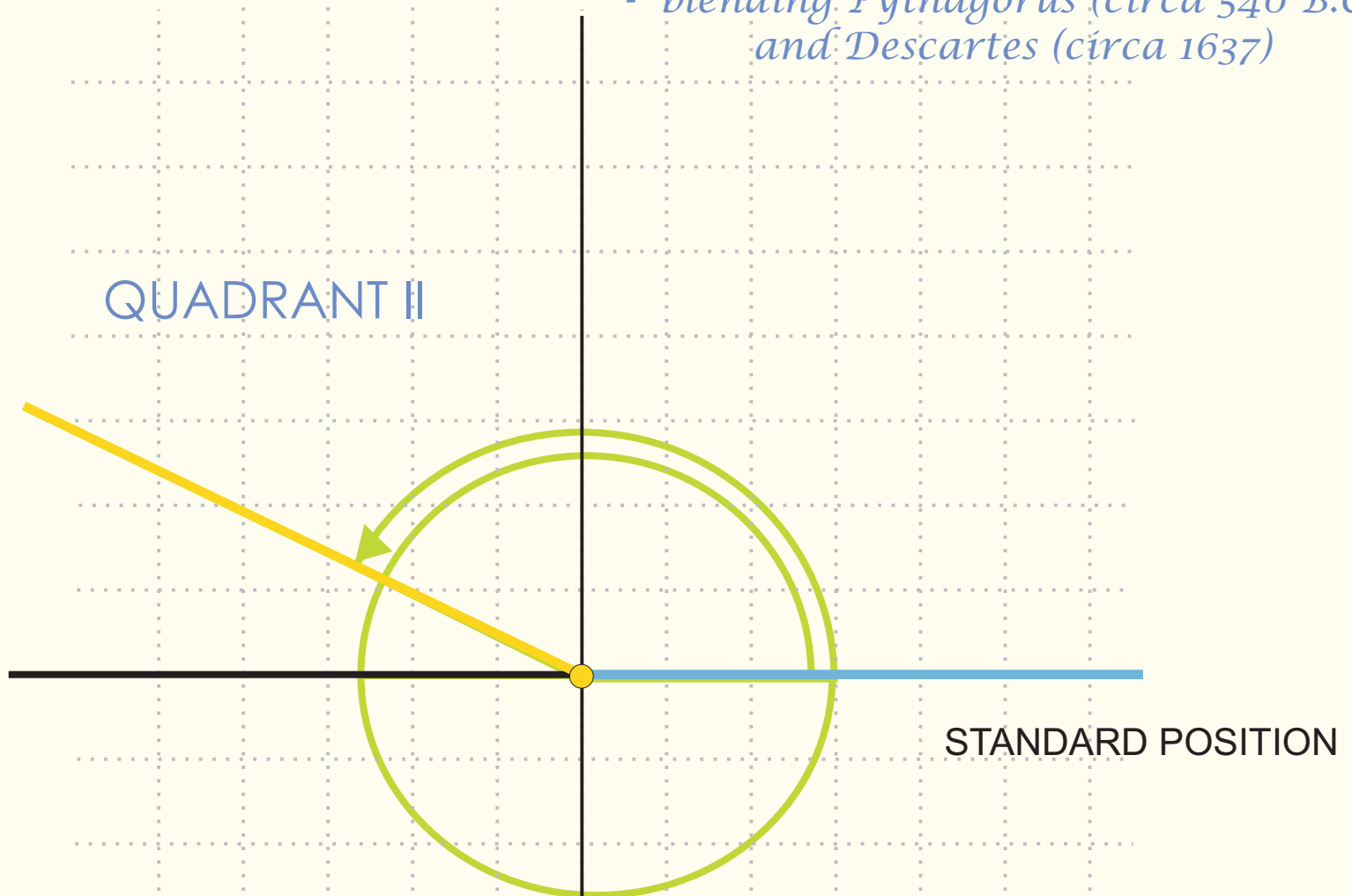
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THE TRIGONOMETRY OF ANGLES IN QUADRANT II

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Quadrant II angles include:

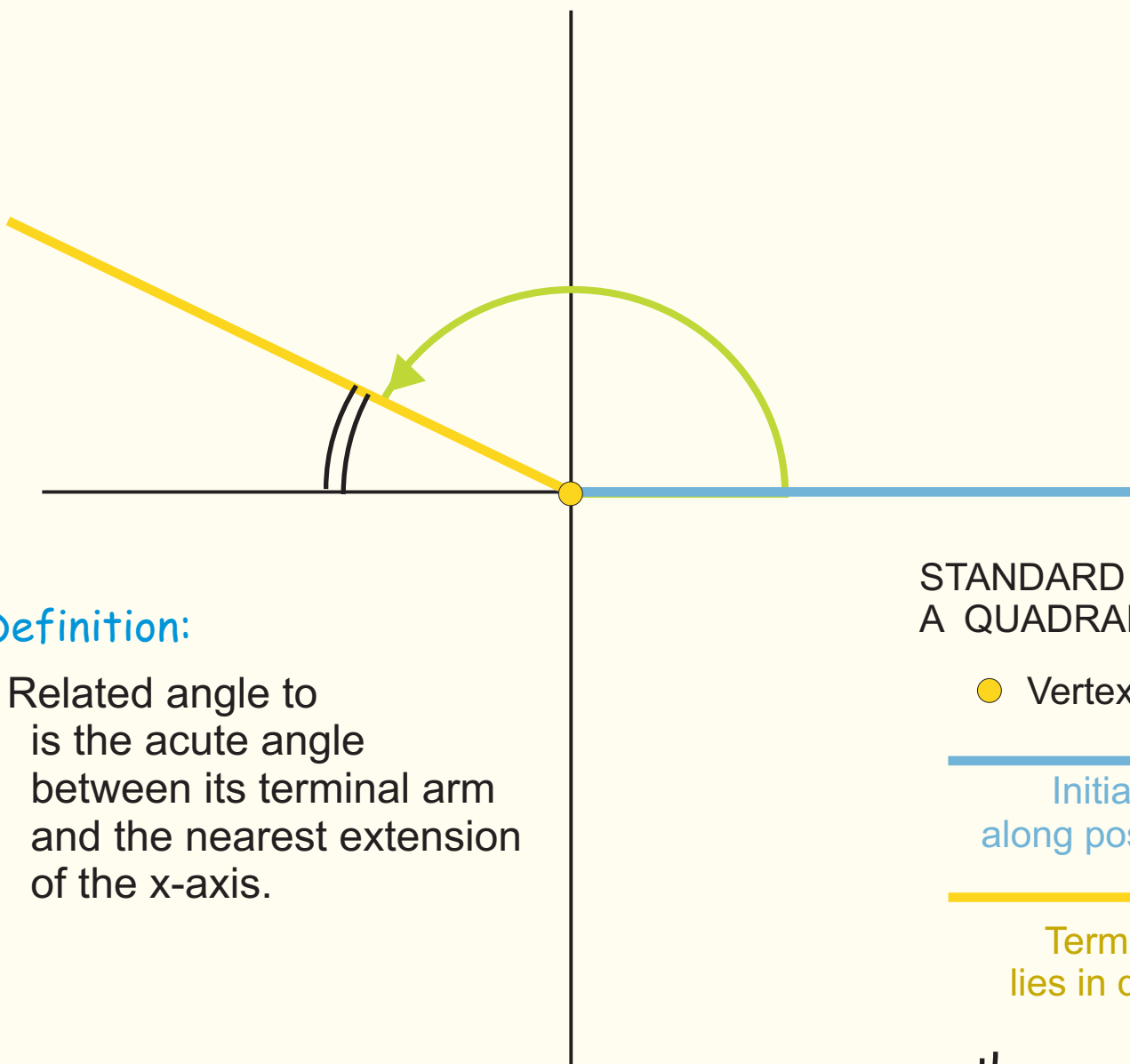
$$120 + 360 = 480 \text{ degrees, or } \frac{2}{3} + 2 = \frac{8}{3} \text{ radians,}$$

$$135 + 360 = 495 \text{ degrees, or } \frac{3}{4} + 2 = \frac{11}{4} \text{ radians,}$$

$$150 + 720 = 870 \text{ degrees, or } \frac{5}{6} + 4 = \frac{29}{6} \text{ radians.}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

- *blending Pythagorus (circa 540 B.C.)
and Descartes (circa 1637)*



Definition:

Related angle to
is the acute angle
between its terminal arm
and the nearest extension
of the x-axis.

STANDARD POSITION OF A QUADRANT II ANGLE

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm
lies in quadrant II

// Related angle
to

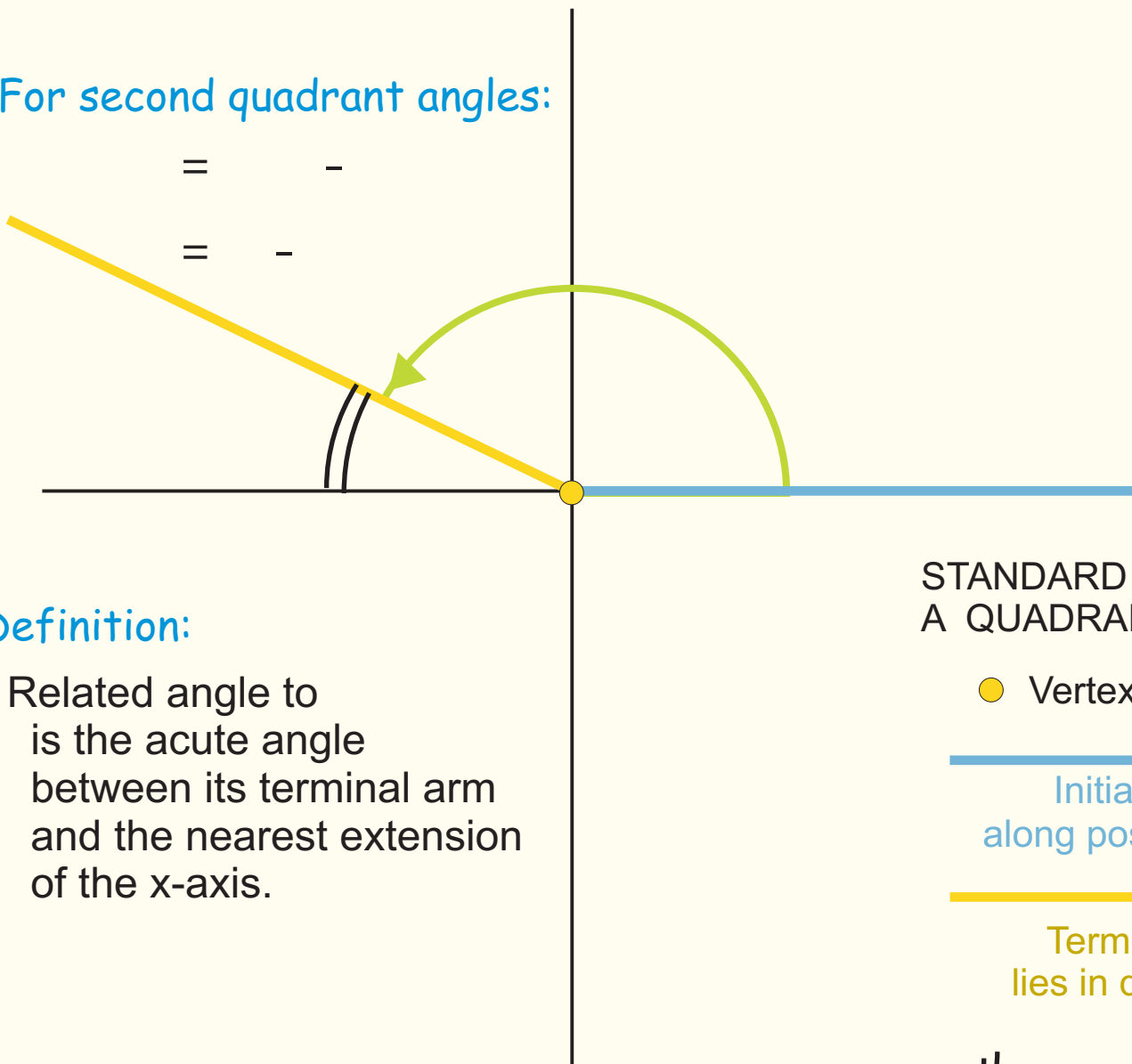
THE TRIGONOMETRY OF ANGLES IN QUADRANT II

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For second quadrant angles:

= -

= -



Definition:

Related angle to
is the acute angle
between its terminal arm
and the nearest extension
of the x-axis.

STANDARD POSITION OF
A QUADRANT II ANGLE

● Vertex at origin

Initial arm
along positive x-axis

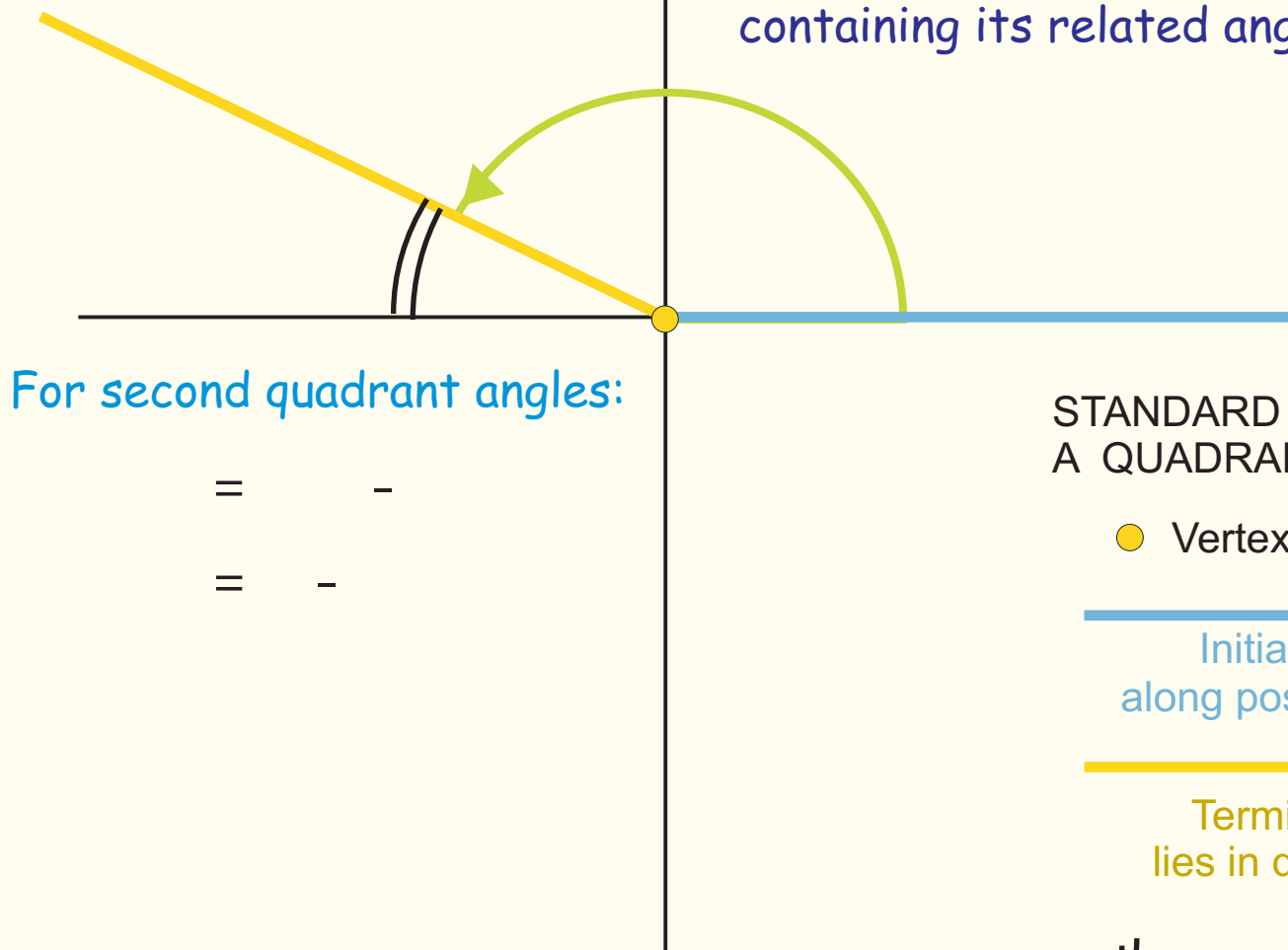
Terminal arm
lies in quadrant II

// Related angle
to

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

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and Descartes (circa 1637)*

The trigonometric measures of
an angle in the SECOND quadrant
are measured using the sides of
the triangle in the SECOND quadrant
containing its related angle .



For second quadrant angles:

$$= -$$

$$= -$$

STANDARD POSITION OF
A QUADRANT II ANGLE

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm
lies in quadrant II

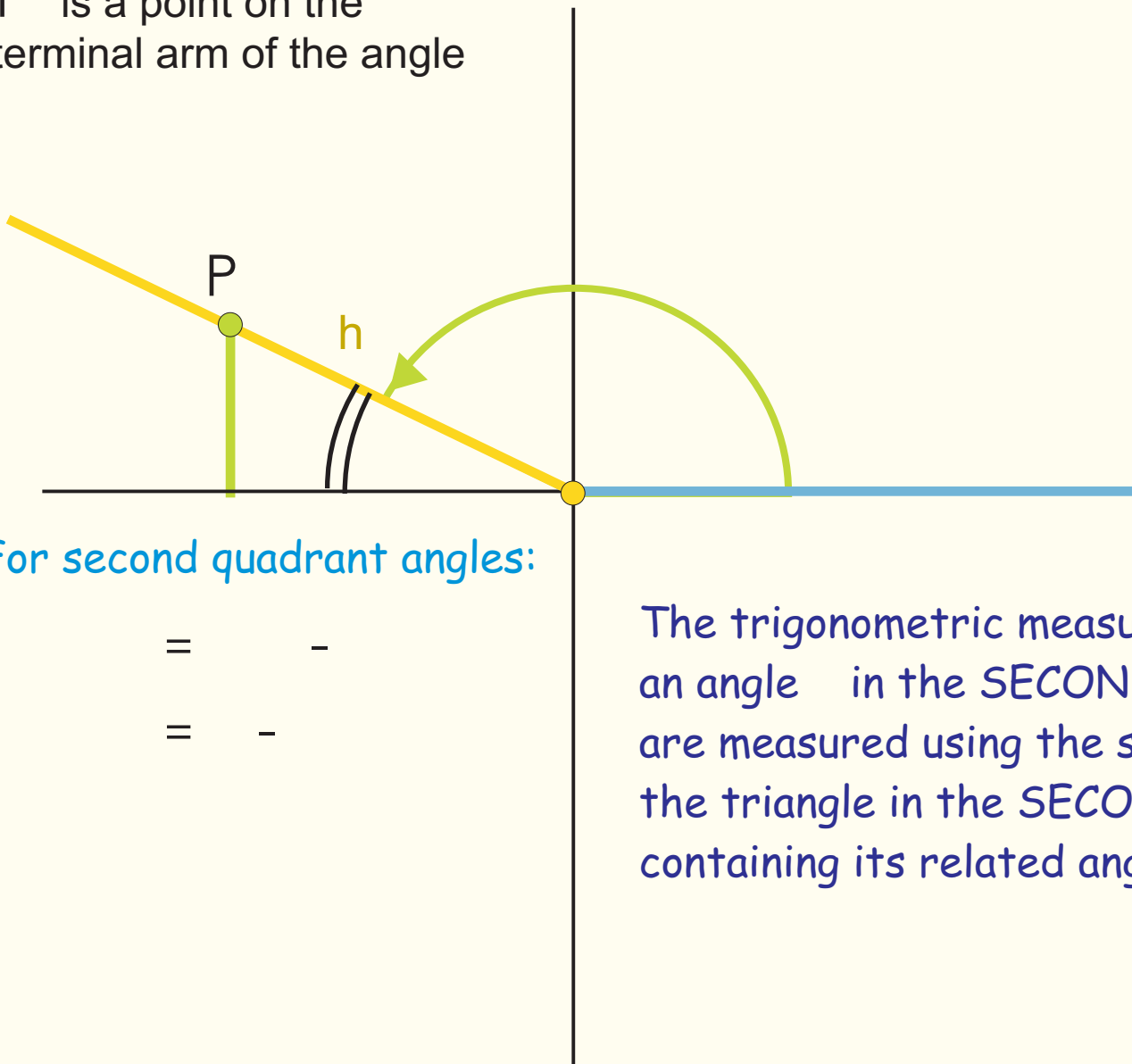
// Related angle
to

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h = distance from P to the origin

P is a point on the
terminal arm of the angle



For second quadrant angles:

= -

= -

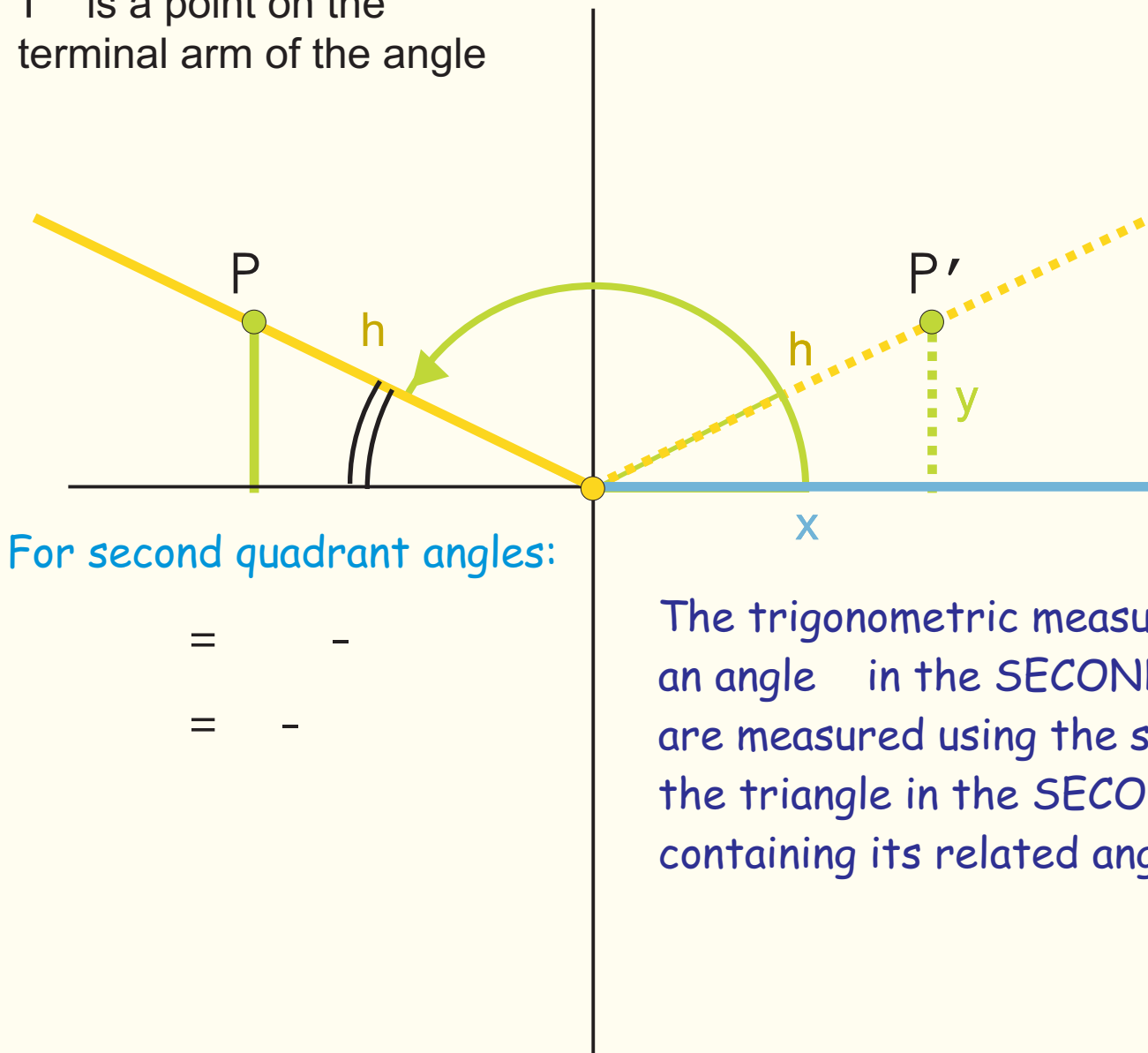
The trigonometric measures of
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are measured using the sides of
the triangle in the SECOND quadrant
containing its related angle .

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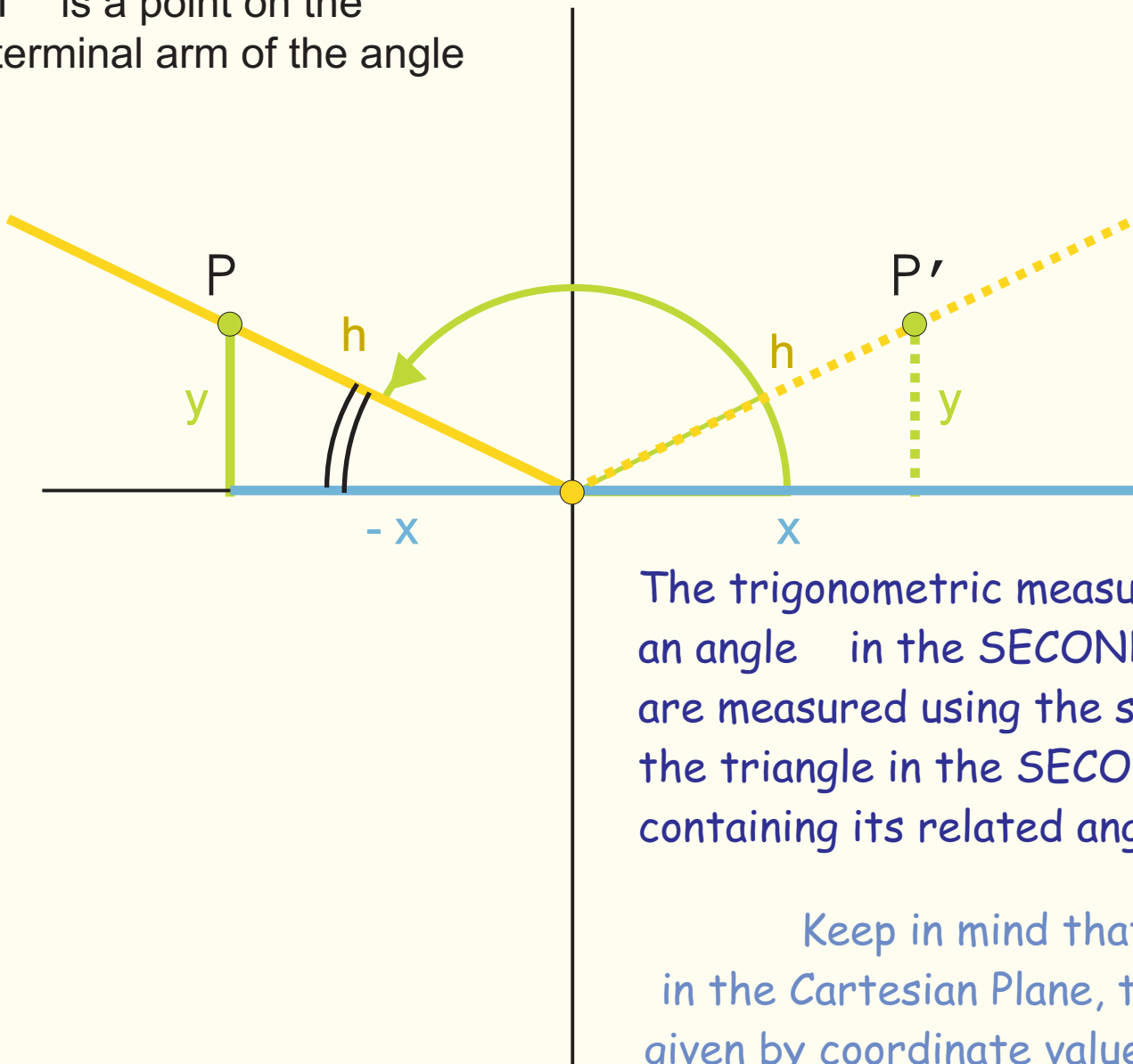
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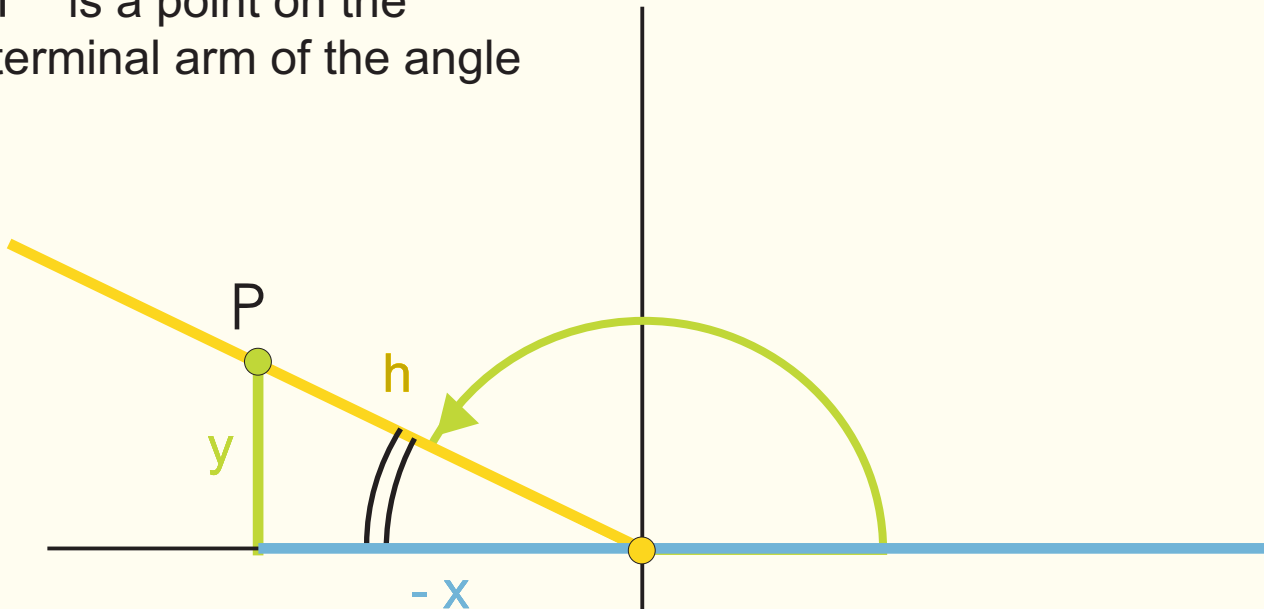
Keep in mind that,
in the Cartesian Plane, the sides
given by coordinate values may be
either positive or negative.

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sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}}$$

The trigonometric measures of
an angle in the **SECOND** quadrant
are measured using the sides of
the triangle in the **SECOND** quadrant
containing its related angle .

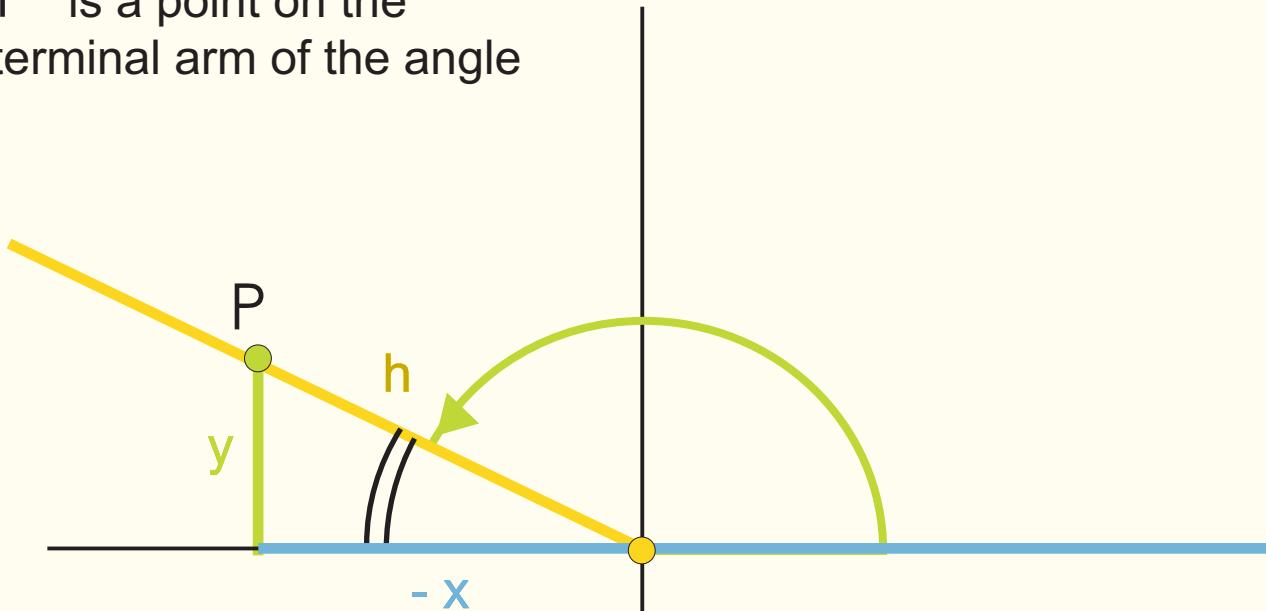
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cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x}$$

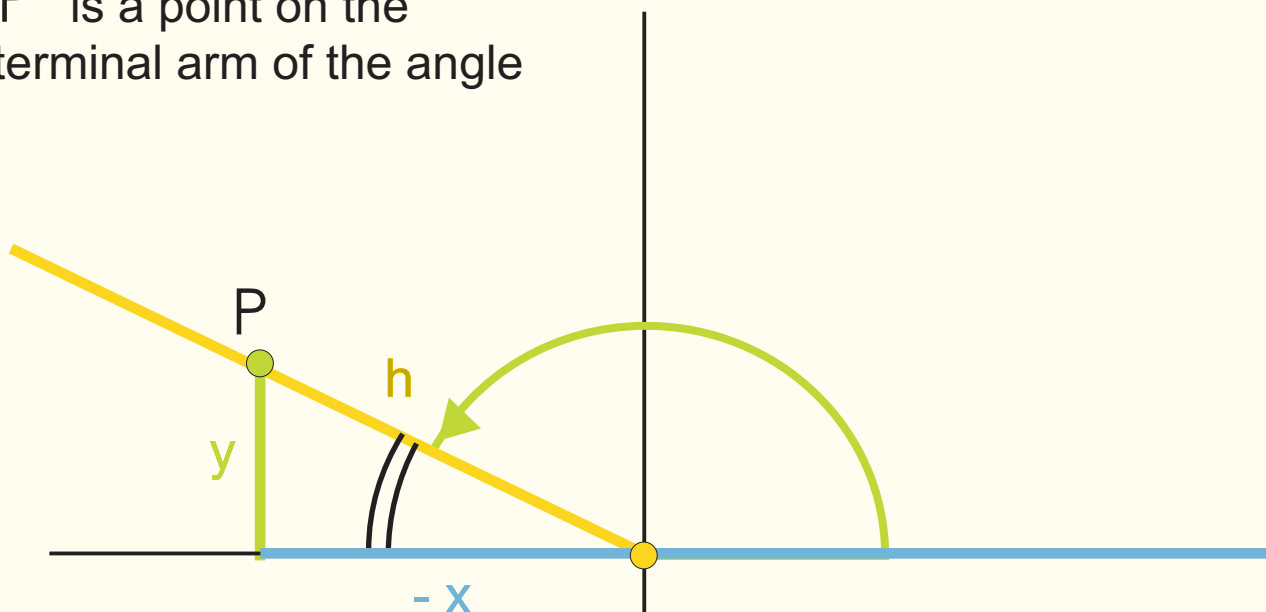
Note that only the Sine and
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have positive measures.

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cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{-x}$$

cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{-x}{y}$$

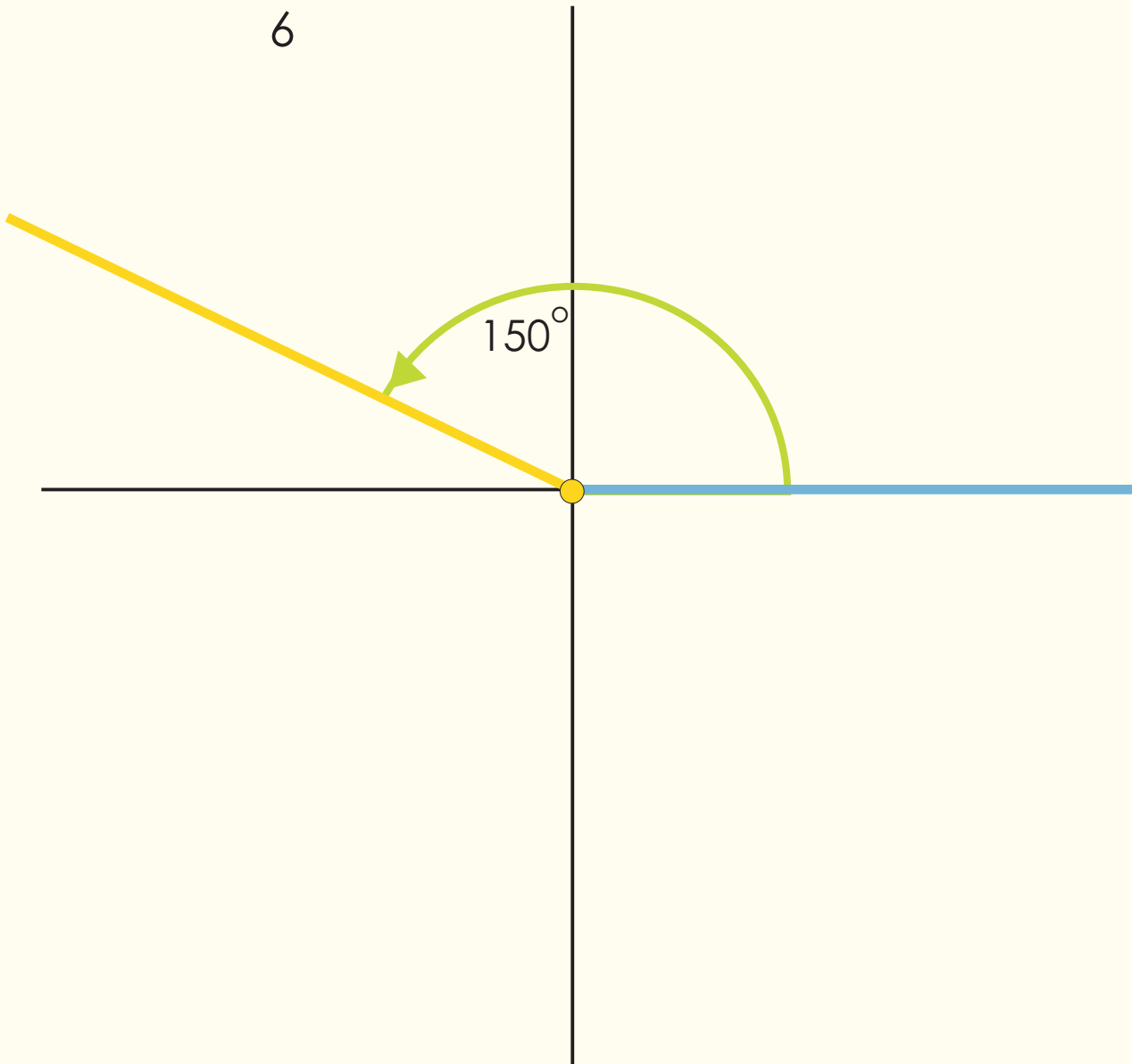
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EXAMPLE

$$= 150^\circ = \frac{5}{6} \text{ radians}$$



Find the sine, cosine and tangent values of $\frac{5}{6}$ radians .

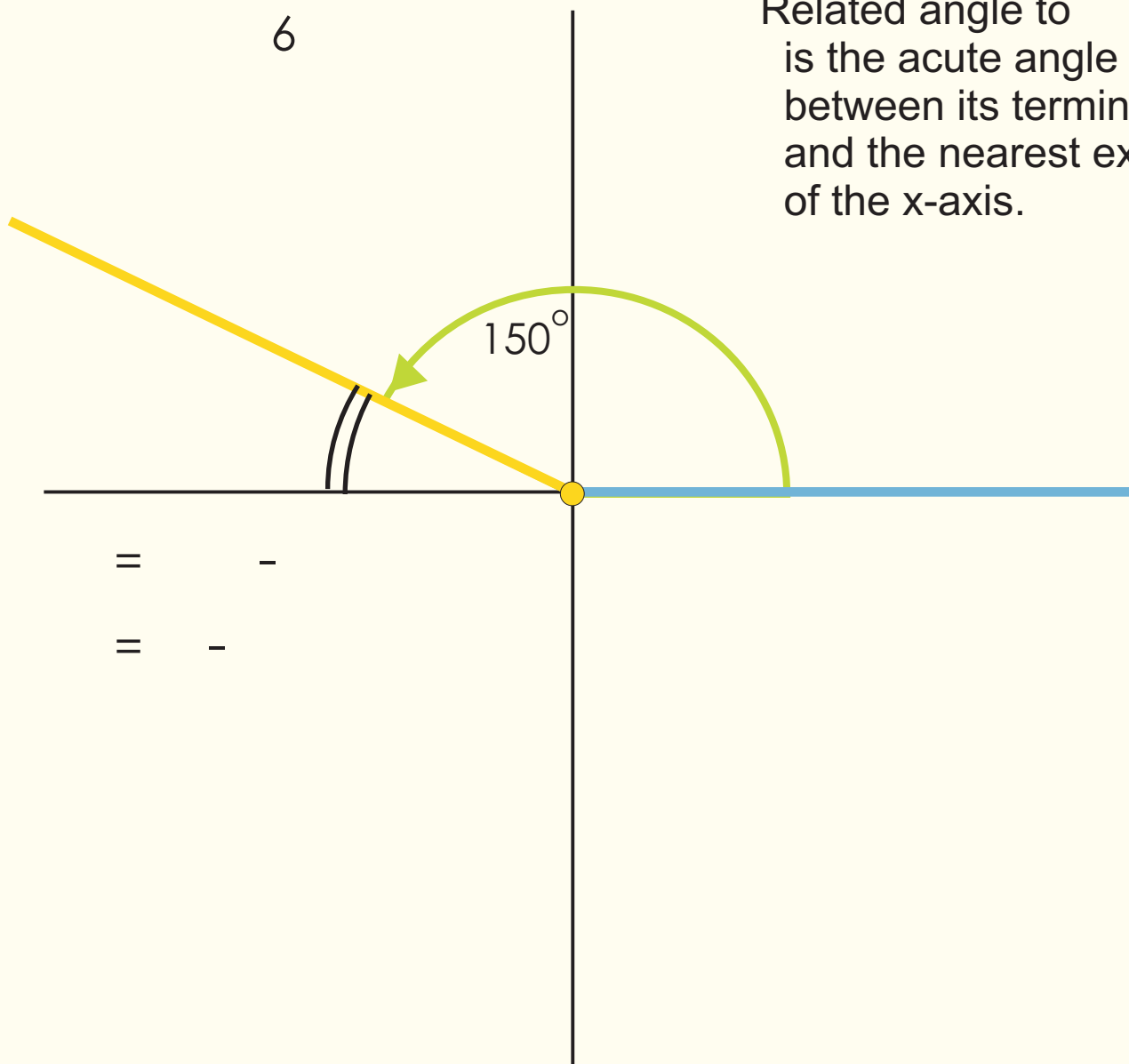
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EXAMPLE

$$= 150^\circ = \frac{5}{6} \text{ radians}$$

Related angle to
is the acute angle
between its terminal arm
and the nearest extension
of the x-axis.



= -

= -

Find the sine, cosine and tangent values of $\frac{5}{6}$ radians .

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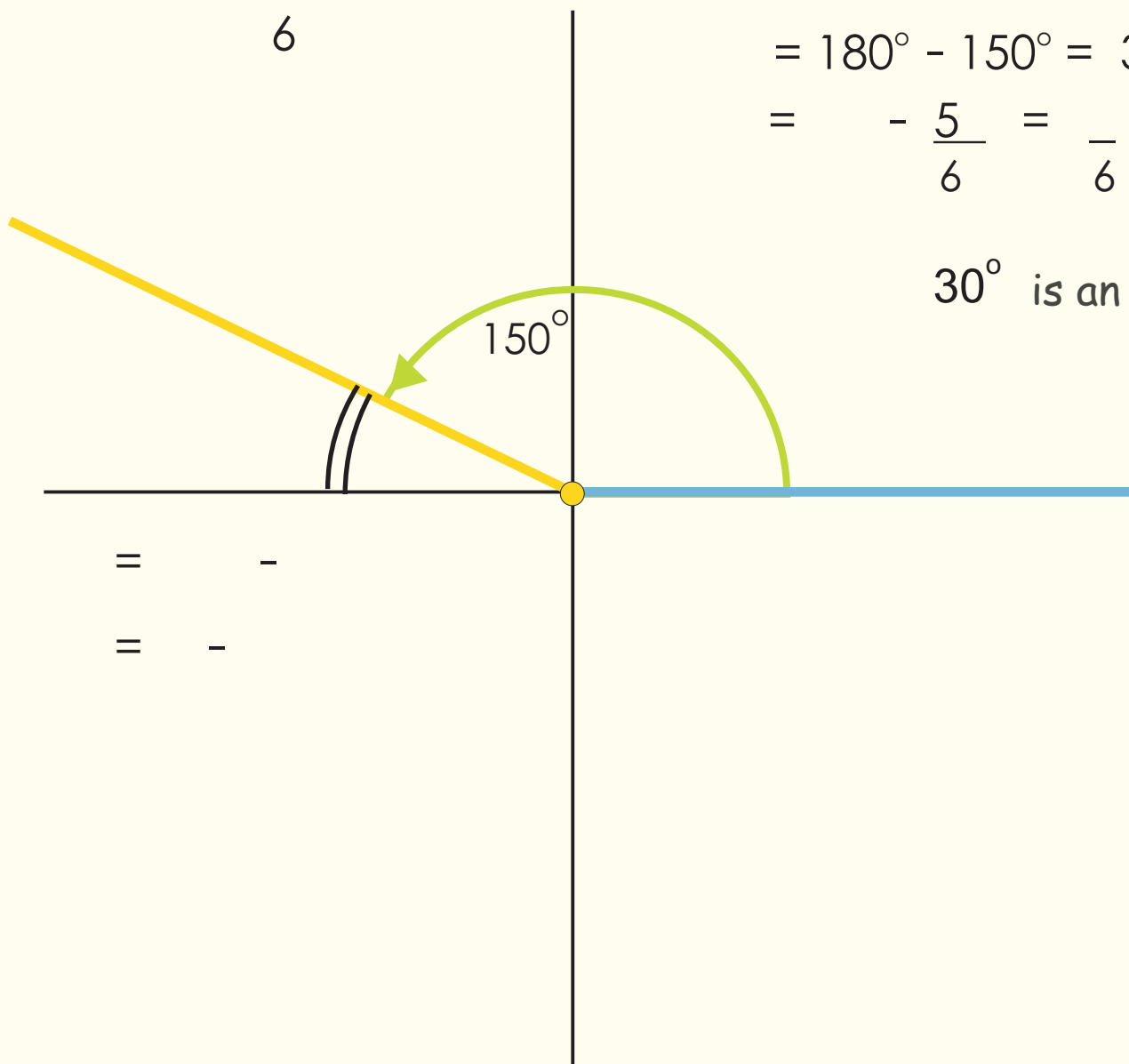
$$= 150^\circ = \frac{5}{6} \text{ radians}$$

$$= -$$

$$= 180^\circ - 150^\circ = 30^\circ$$

$$= - \frac{5}{6} = - \frac{5}{6} \text{ radians}$$

30° is an acute angle.



$$= -$$

$$= -$$

Find the sine, cosine and tangent values of $\frac{5}{6}$ radians .

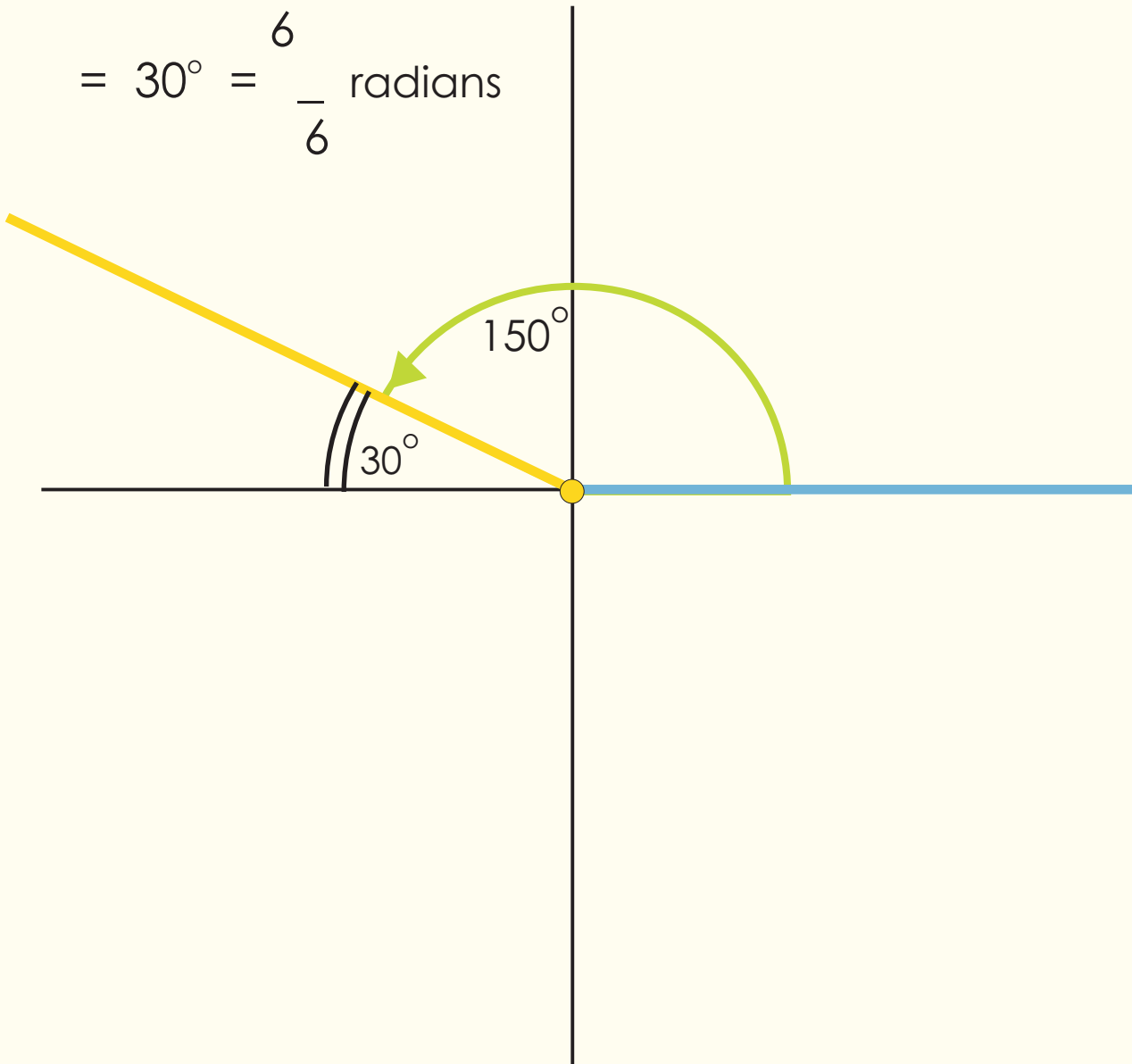
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$$= 150^\circ = \frac{5}{6} \text{ radians}$$

$$= 30^\circ = \frac{1}{6} \text{ radians}$$



Find the sine, cosine and tangent values of $\frac{5}{6}$ radians .

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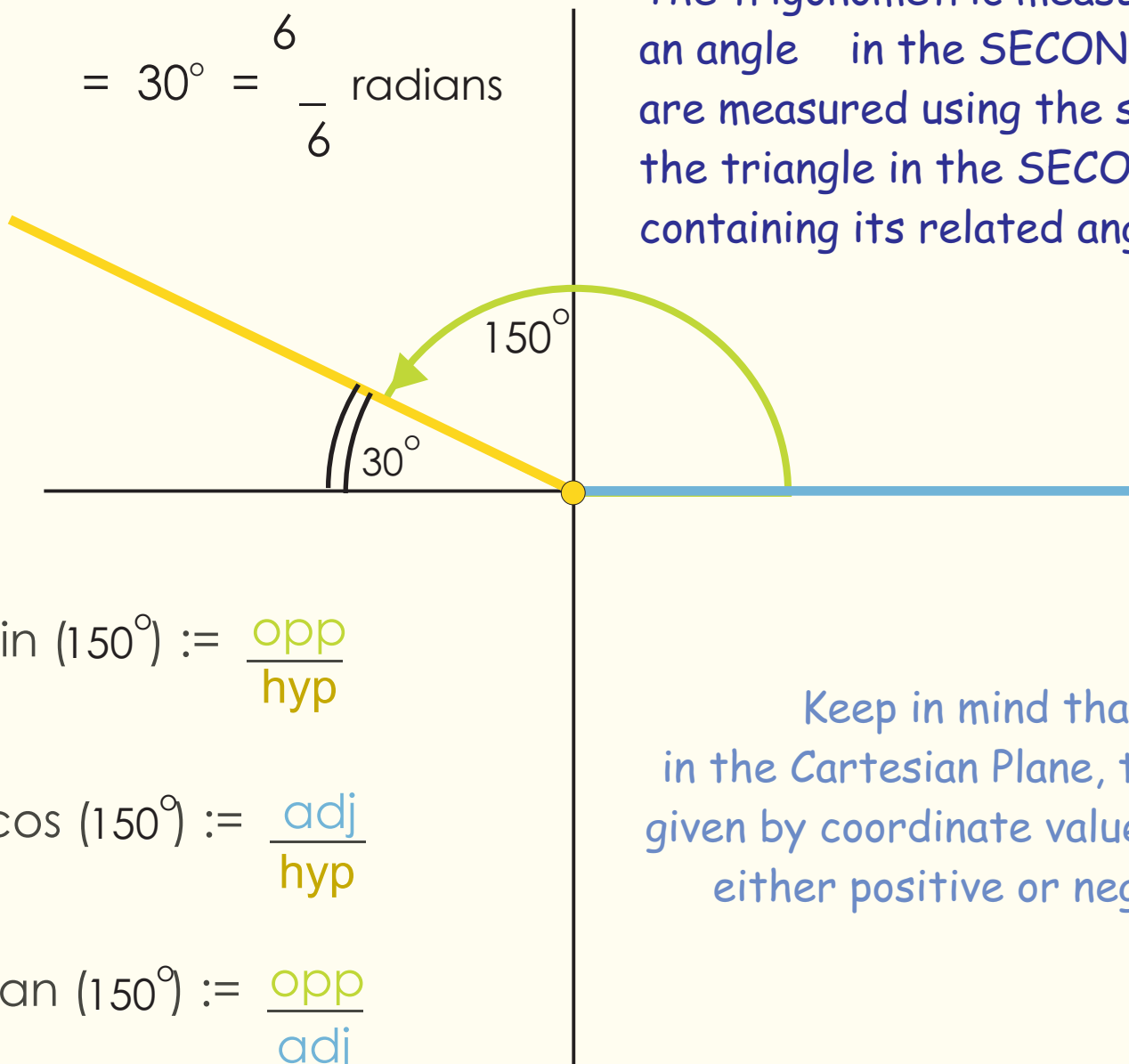
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EXAMPLE

$$= 150^\circ = \frac{5}{6} \text{ radians}$$

$$= 30^\circ = \frac{1}{6} \text{ radians}$$

The trigonometric measures of an angle in the SECOND quadrant are measured using the sides of the triangle in the SECOND quadrant containing its related angle.



$$\sin (150^\circ) := \frac{\text{opp}}{\text{hyp}}$$

$$\cos (150^\circ) := \frac{\text{adj}}{\text{hyp}}$$

$$\tan (150^\circ) := \frac{\text{opp}}{\text{adj}}$$

Keep in mind that,
in the Cartesian Plane, the sides
given by coordinate values may be
either positive or negative.

THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

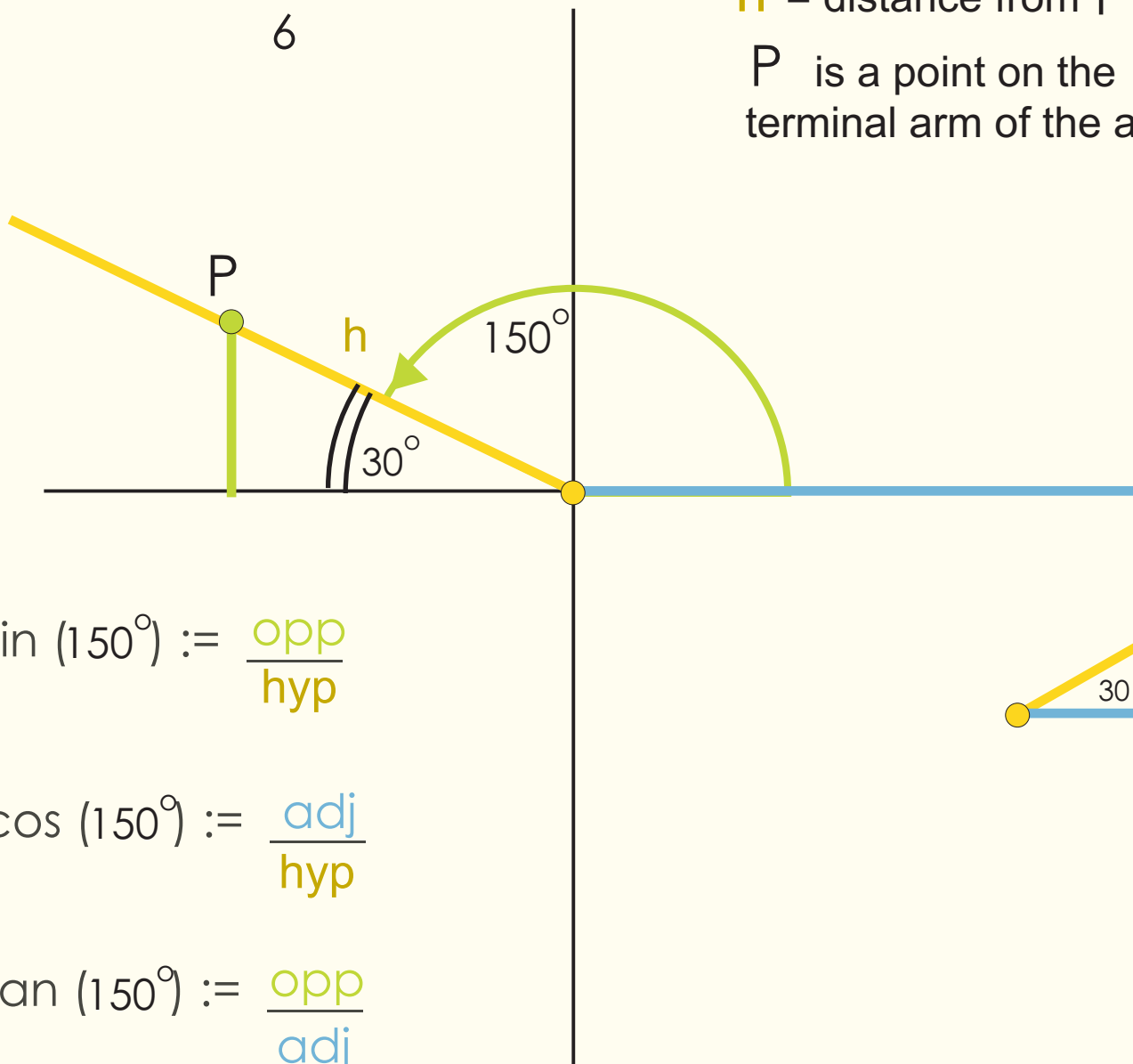
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$$= 150^\circ = \frac{5}{6} \text{ radians}$$

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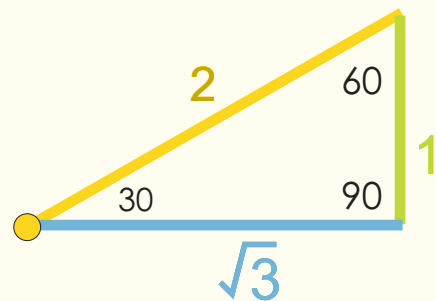
P is a point on the
terminal arm of the angle



$$\sin (150^\circ) := \frac{\text{opp}}{\text{hyp}}$$

$$\cos (150^\circ) := \frac{\text{adj}}{\text{hyp}}$$

$$\tan (150^\circ) := \frac{\text{opp}}{\text{adj}}$$



30°

$$\begin{aligned} \text{hyp} &= 2 \\ \text{opp} &= 1 \\ \text{adj} &= \sqrt{3} \end{aligned}$$

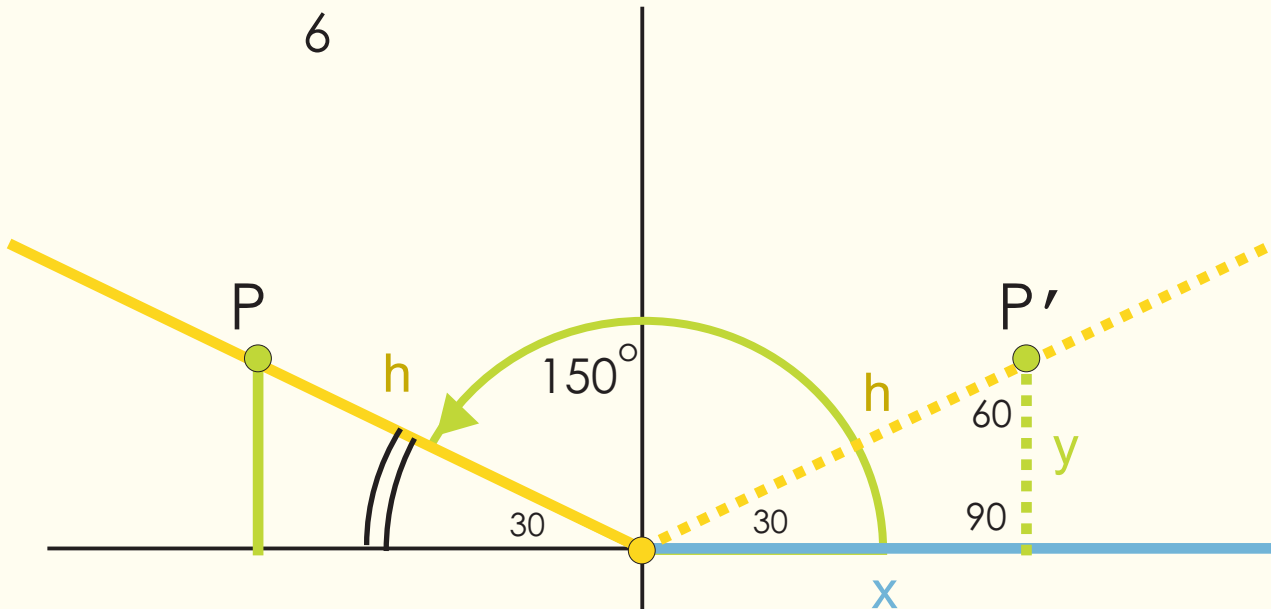
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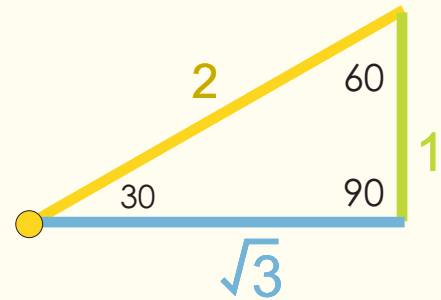
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$$\sin (150^\circ) := \frac{\text{opp}}{\text{hyp}}$$

$$\cos (150^\circ) := \frac{\text{adj}}{\text{hyp}}$$

$$\tan (150^\circ) := \frac{\text{opp}}{\text{adj}}$$



$$\begin{aligned} 30^\circ \\ \text{hyp} &= 2 \\ \text{opp} &= 1 \\ \text{adj} &= \sqrt{3} \end{aligned}$$

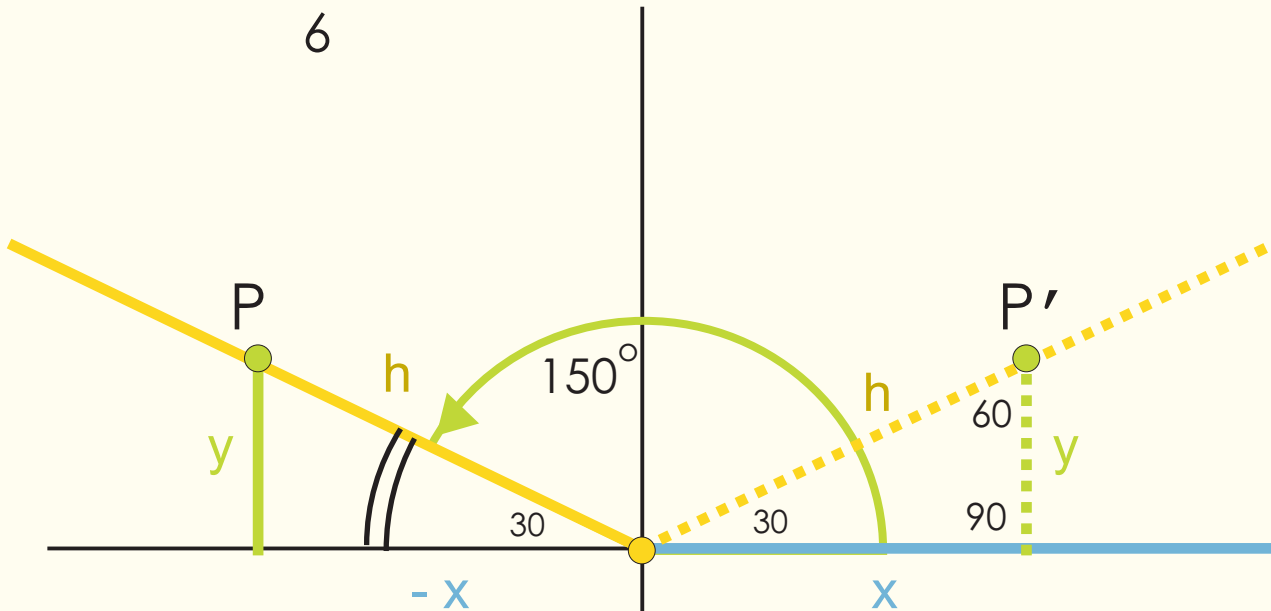
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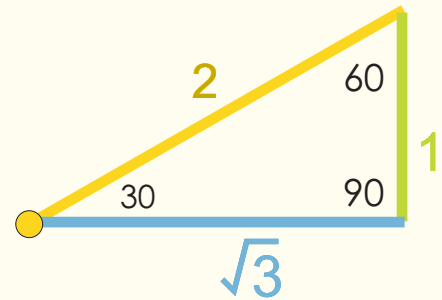
$$= 150^\circ = \frac{5}{6} \text{ radians}$$



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$$\cos (150^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h}$$

$$\tan (150^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x}$$



30°

hyp = 2
opp = 1
adj = $\sqrt{3}$

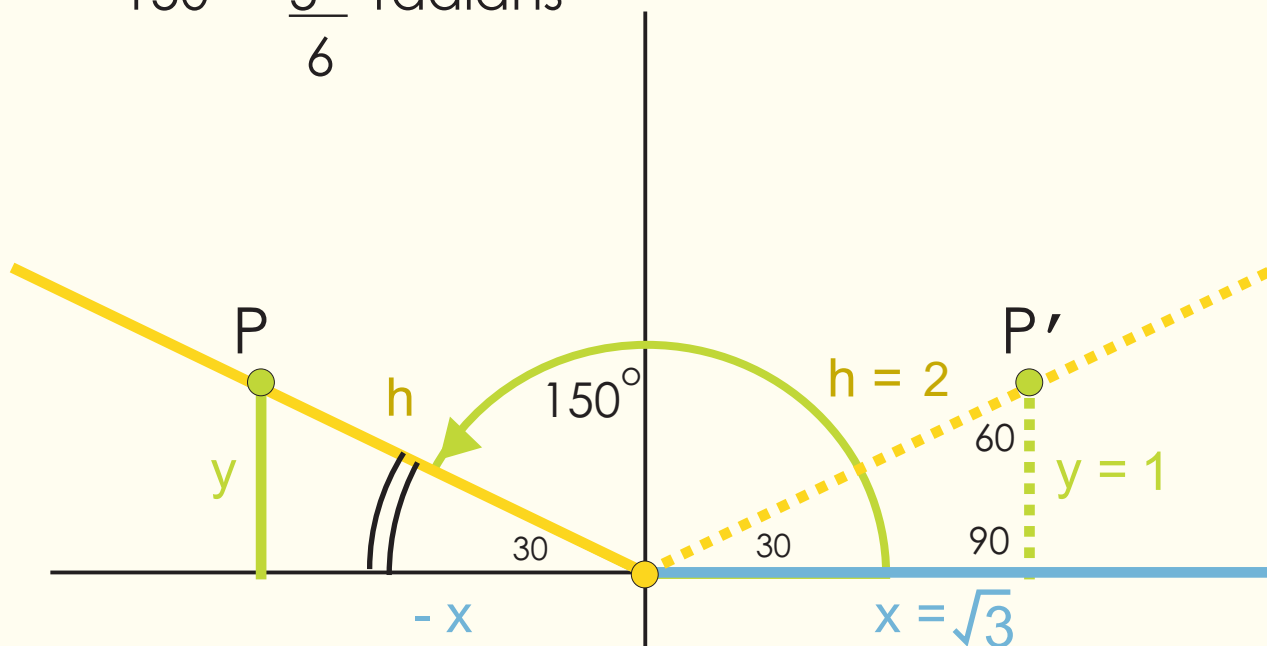
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EXAMPLE

$$= 150^\circ = \frac{5}{6} \text{ radians}$$



$$\sin (150^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h} = \frac{1}{2}$$

$$\cos (150^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-\sqrt{3}}{2}$$

$$\tan (150^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x} = \frac{1}{-\sqrt{3}}$$

$$\begin{array}{l} 150^\circ \\ \text{hyp} = 2 \\ \text{opp} = 1 \\ \text{adj} = -\sqrt{3} \end{array}$$

$$\begin{array}{l} 30^\circ \\ \text{hyp} = 2 \\ \text{opp} = 1 \\ \text{adj} = \sqrt{3} \end{array}$$

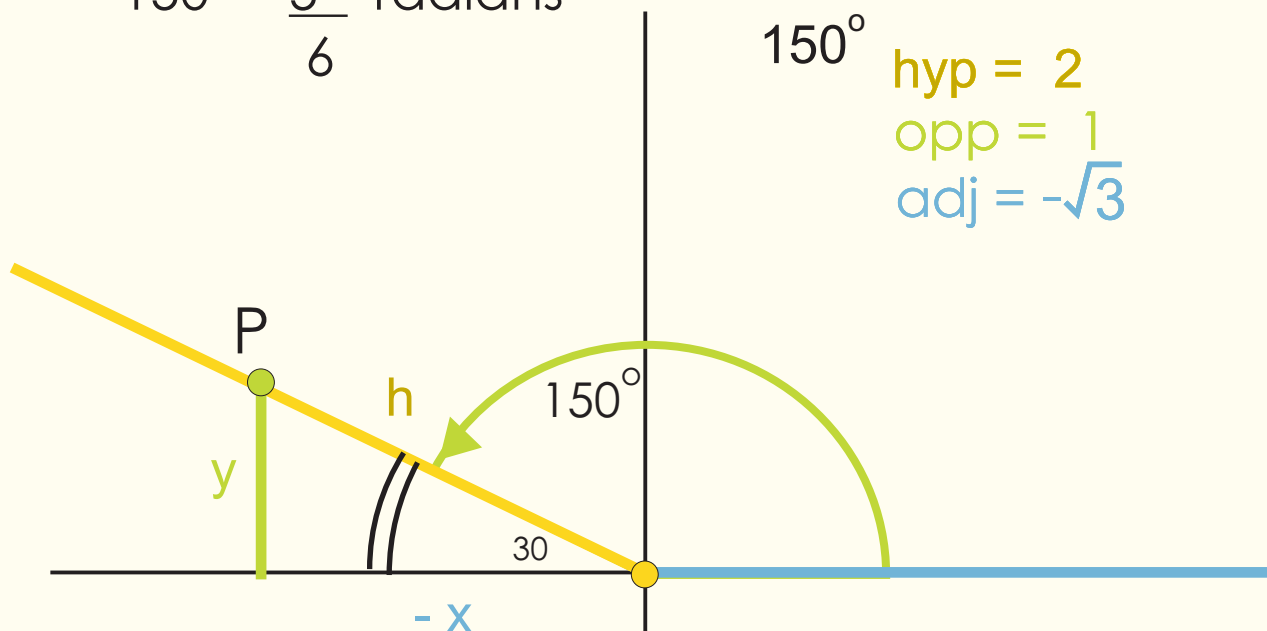
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EXAMPLE

$$= 150^\circ = \frac{5}{6} \text{ radians}$$



$$\sin (150^\circ) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h} = \frac{1}{2}$$

$$\cos (150^\circ) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-\sqrt{3}}{2}$$

$$\tan (150^\circ) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x} = \frac{1}{-\sqrt{3}}$$

Note that only the Sine and its reciprocal, the Cosecant, of an angle in the SECOND quadrant have positive measures.

THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

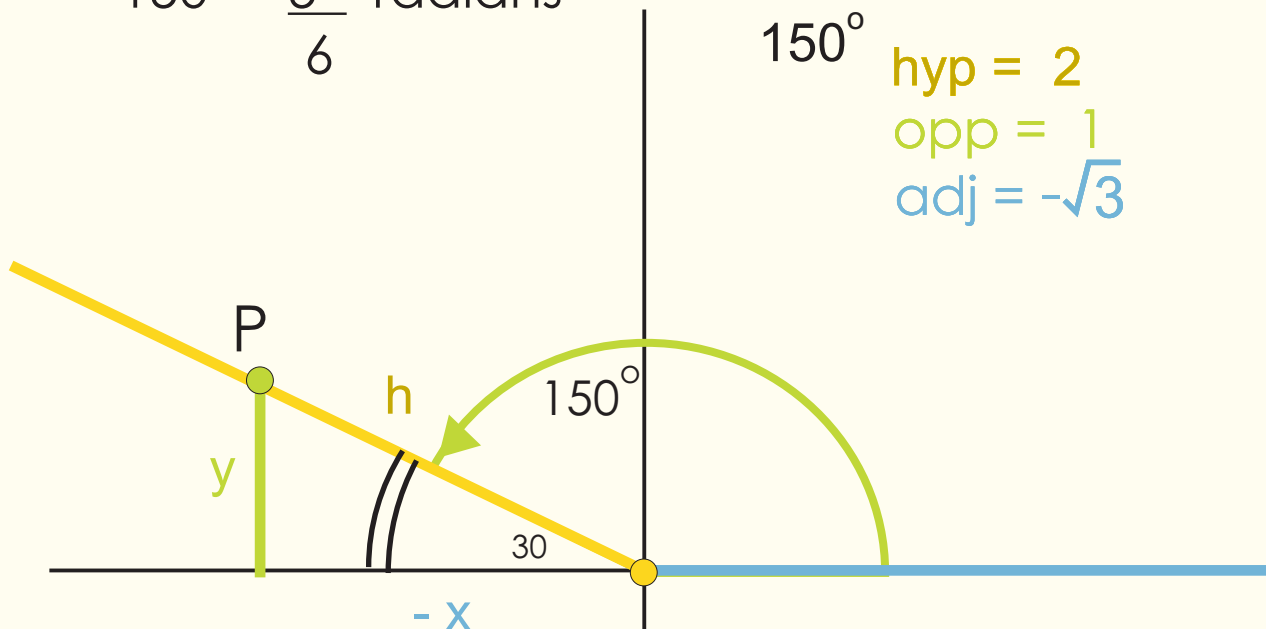
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EXAMPLE

$$= 150^\circ = \frac{5}{6} \text{ radians}$$

$$\begin{aligned} 150^\circ \quad & \text{hyp} = 2 \\ & \text{opp} = 1 \\ & \text{adj} = -\sqrt{3} \end{aligned}$$

$$\begin{aligned} 30^\circ \quad & \text{hyp} = 2 \\ & \text{opp} = 1 \\ & \text{adj} = \sqrt{3} \end{aligned}$$



$$\csc(150^\circ) = \frac{\text{hyp}}{\text{opp}} = \frac{h}{y} = \frac{2}{1}$$

$$\sec(150^\circ) = \frac{\text{hyp}}{\text{adj}} = \frac{h}{-x} = \frac{2}{-\sqrt{3}}$$

$$\cot(150^\circ) = \frac{\text{adj}}{\text{opp}} = \frac{-x}{y} = \frac{-\sqrt{3}}{1}$$

Note that only the Sine and its reciprocal, the Cosecant, of an angle in the SECOND quadrant have positive measures.

THE TRIGONOMETRIC MEASURES OF 150°

QUADRANT II

$\sin (\)$ positive

$\cos (\)$ negative

$\tan (\)$ negative

QUADRANT I

$\sin (\)$ positive

$\cos (\)$ positive

$\tan (\)$ positive

QUADRANT III

QUADRANT IV

